5.2 Videos Guide

5.2a

- Disk Method: The volume of a solid obtained by rotating the graph of a function f, $a \le x \le b$ about the x-axis:
  - $V = \int_a^b \pi[f(x)]^2 \, dx$
- Washer Method: The volume of a solid obtained by rotating the region between two functions, f and g, $a \le x \le b$ (for $f \ge g$ over the interval $[a, b]$) about the x-axis:
  - $V = \int_a^b \pi\{[f(x)]^2 - [g(x)]^2\} \, dx$

  In both cases, the representative rectangle (represented as dx in these cases) is *perpendicular* to the axis of rotation. (Analogous formulas exist for functions of y.) Note that these are based on the area of a circle.

5.2b

Exercises:

- Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.
  - $y = \frac{1}{x}, \;\; y = 0, \;\; x = 1, \;\; x = 4;$ rotate about the x-axis
  - $x = 2 - y^2, \;\; x = y^4;$ rotate about the y-axis

5.2c

- Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.
  $y = x^3, \;\; y = 1, \;\; x = 2;$ rotate about $y = -3$
- When the axis of rotation is not a coordinate axis, we can generalize the volume of a solid of revolution as follows. If R is the distance between f and the axis of rotation and r is the distance between g and the axis of rotation, the above formulas become
  - $V = \pi \int_a^b R^2 \, dx$
  - $V = \pi \int_a^b (R^2 - r^2) \, dx$

- Various possible disk and washer setups—focus on identifying R and r

5.2d

Definition: (volume—by slicing)

- Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the volume of S is

  $$V = \int_a^b A(x) \, dx$$

Exercises:

- Find the volume of the described solid S.
  - The base of S is a circular disk with radius r. Parallel cross-sections perpendicular to the base are squares.

5.2e

  - The base of S is the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$. Cross-sections perpendicular to the y-axis are equilateral triangles.